AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

Note on Problem 33. By Henry Heaton, Atlantic, Iowa.

The result obtained in the published solution of this problem cannot be correct.

The area of the regular pentagon is $3.6327a^2$. The area of each of the infinite number of regular polygons whose apothem is a and number of sides greater than five, is less than $3.6327a^2$, while that of only two, the square and triangle, is greater. Hence the average area of all regular polygons with apothem a is less than $3.6327a^2$. Hence the result obtained in the published solution ($3.8693a^2$) is too large.

In a similar manner it may be shown that any result larger than $a^2\pi$ is too large, while it is evident that any result smaller than that is too small.

37. Proposed by HENRY HEATON, M. Sc., Atlantic, Iowa.

Required the average area of all triangles two of whose sides are a and b.

I. Solution by the PROPOSER.

It is well known that every triangle consists of six parts, three sides and three angles, and one side with any two other parts determines the triangle.

In constructing this triangle we may use all possible values, first, of the included angle C, second, of the third side, c, third, of the angle A, and fourth, of the angle B. This gives us four cases.

I. Put angle $C=\theta$. Then $A_1=\dfrac{ab}{2}\displaystyle\int_0^\pi \sin\theta d\theta \div \int_0^\pi d\theta=\frac{ab}{\pi}$.

II. Put side $c=x$. Then $A_2=\frac{1}{4}\displaystyle\int_{a-b}^{a+b}[(a+b)^2-x^2]^{\frac{1}{2}}[x^2-(a-b)^2]^{\frac{1}{2}}dx$.

$$\div\int_{a-b}^{a+b}dx=\frac{a+b}{12b}\left\{(a^2+b^2)E\left[\left(\frac{2\sqrt{ab}}{a-b}\right),\tfrac{1}{2}\pi\right]-(a-b)^2F\left[\left(\frac{2\sqrt{ab}}{a+b}\right),\tfrac{1}{2}\pi\right]\right\}.$$

(To integrate this expression put $x=[(a+b)^2-4ab\sin\theta]^{\frac{1}{2}}$).

III. Put angle $A=\theta$, b being $<a$, then

$$A_3=\tfrac{1}{2}b\int_0^\pi[b\cos\theta+(a^2-b^2\sin^2\theta)^{\frac{1}{2}}]\sin\theta d\theta\div\int_0^\pi d\theta=\frac{ab}{2\pi}+\left(\frac{a^2-b^2}{4\pi}\right)\log_e\left(\frac{a+b}{a-b}\right).$$